Belotti, P., Kirches, C., Leyffer, S., Linderoth, J., Luedtke, J., & Mahajan, A. (2013). Mixed-integer nonlinear optimization. *Acta Numerica, 22,* 1-131. doi: 10.1017/S0962492913000032

Abstract

Many optimal decision problems in scientific, engineering, and public sector applications involve both discrete decisions and nonlinear system dynamics that affect the quality of the final design or plan. These decision problems lead to mixed-integer nonlinear programming (MINLP) problems that combine the combinatorial difficulty of optimizing over discrete variable sets with the challenges of handling nonlinear functions. We review models and applications of MINLP, and survey the state of the art in methods for solving this challenging class of problems.

Most solution methods for MINLP apply some form of tree search. We distinguish two broad classes of methods: single-tree and multitree methods. We discuss these two classes of methods first in the case where the underlying problem functions are convex. Classical single-tree methods include nonlinear branch-and-bound and branch-and-cut methods, while classical multitree methods include outer approximation and Benders decomposition. The most efficient class of methods for convex MINLP are hybrid methods that combine the strengths of both classes of classical techniques.

Non-convex MINLPs pose additional challenges, because they contain non-convex functions in the objective function or the constraints; hence even when the integer variables are relaxed to be continuous, the feasible region is generally non-convex, resulting in many local minima. We discuss a range of approaches for tackling this challenging class of problems, including piecewise linear approximations, generic strategies for obtaining convex relaxations for non-convex functions, spatial branch-and-bound methods, and a small sample of techniques that exploit particular types of non-convex structures to obtain improved convex relaxations.

We finish our survey with a brief discussion of three important aspects of MINLP. First, we review heuristic techniques that can obtain good feasible solution in situations where the search-tree has grown too large or we require real-time solutions. Second, we describe an emerging area of mixed-integer optimal control that adds systems of ordinary differential equations to MINLP. Third, we survey the state of the art in software for MINLP. (317 words)

Carella, N. A. (2015). Least prime primitive roots. *International Journal of Mathematics and Computer Science, 10*(2), 185-194.

Abstract

This note presents an upper estimate of the least prime primitive roots $g_*(p)$ modulo p, a large prime. The current literature has several estimates of the least prime primitive root $g_*(p)$ modulo a prime p 2 such as $g_*(p)$ pc, c > 2.8. The estimate provided within seems to sharpen this estimate to the smaller estimate $g_*(p)$ p5/log logp uniformly for all large primes p 2.